# Spatial Power Splitting and Combining Based on the Talbot Effect

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***Abstract*—The Talbot effect, a multimode interference phenomenon, is investigated as a technique for combining power from solid-state devices in order to generate higher levels of microwave and millimeter-wave power in a process referred to as quasioptical or spatial power combining. We explore the feasibility of using the Talbot effect to implement a 1 $\times$ 8 power splitter and an 8 $\times$ 1 power combiner at 94 GHz. We report the first demonstration of the multimode interface phenomenon in a planar waveguide at 8 GHz.**

***Index Terms*—Coupled mode analysis, interference, MMIC power amplifiers, multimode waveguides, power combiners, power dividers.**

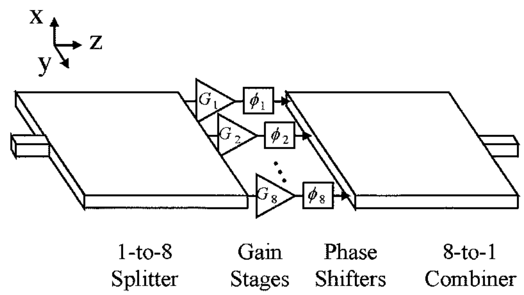


Fig. 1. Architecture for a two-dimensional spatial power combining amplifier.

## I. Introduction

QUASI-OPTICAL or spatial power combining systems generate higher levels microwave and millimeter-wave power by summing the outputs from an array of solid-state devices into a single propagating mode. This is accomplished with lower losses and potentially reduced matching sensitivity than achievable with transmission lined based combining [1]. Recent developments in spatial power combining systems merge solid-state amplifiers with power splitting and combining in planar dielectric slab waveguides [2], [3]. Waveguide lensing elements perform the power splitting and combining functions. However, these elements introduce up to 6 dB of the total insertion loss, which reduces the efficiency of the passive structure. A possible solution to this problem is based on the Talbot effect [4]. The Talbot effect has been used extensively in 0.83 to 1.55 $\mu$m integrated optics as a lensless imaging technique for efficient power splitting, combining, and routing [5]–[7].

In this letter, we investigate the application of the Talbot effect to planar waveguide structures for low loss power splitting and combining in the microwave regime. We have designed and simulated a 1 $\times$ 8 power splitter and an 8 $\times$ 1 power combiner in a quartz waveguide operating at 94 GHz. We experimentally demonstrate for the first time the multimode interference (MMI) phenomenon at microwave frequencies.

Manuscript received June 22, 2001; revised October 19, 2001. This material is based upon work supported in part by the U.S. Army Research Laboratory and the U.S. Army Research Office under Grant DAAG55-98-1-0440. The review of this lettter was arranged by Associate Editor Dr. Arvind Sharma.

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Publisher Item Identifier S 1531-1309(02)00863-2.

## II. The Talbot Effect in Quasi-Optical Slab Waveguides

The system concept for a quasioptical power amplifier is shown in Fig. 1. The Talbot effect provides an efficient means of splitting an input field distribution into $N$ equal intensity outputs. Each output can then be amplified using solid-state waveguide amplifiers. Given the appropriate phase shifts among each of the $N$ channels, the Talbot effect can again be used to image the amplified fields into a single propagating mode.

When applied to planar guided-wave structures, Talbot self-imaging is conveniently described as a multimode interference phenomenon. Consider the 1 $\times$ 8 splitter shown on the left in Fig. 1. An input rectangular waveguide, which supports a single TE mode, center-feeds a wider rectangular waveguide, which supports multiple lateral TE modes (but only a single transverse mode). At $z = 0$, the single input mode is decomposed into the $\nu$ lateral modes supported in the MMI region. Since each of the lateral modes propagates through the MMI region with a different phase velocity, a phase difference among the modes accumulates. At the plane along the $z$-axis where the accumulated phase differences reach an integer multiple of $2\pi$, an image (called the self-image) of the input field distribution is formed.

In the analysis of the MMI phenomenon, we reduce the 3-dimensional electromagnetic field problem into a 2-dimensional problem by assuming that the modes have the same transverse behavior everywhere within the MMI region. The field profile at an arbitrary position $z$ may be represented as a superposition of the $\nu$ lateral modes supported in the MMI region

$$\Psi(y,\, z) = \sum_{m=0}^{\nu-1} c_m \phi_m(y) \exp(-j\beta_m z) \tag{1}$$

where the $m$th mode is described by a field excitation coefficient, $c_m$, its field distribution profile, $\phi_m(y)$, and its propagation constant, $\beta_m$. These lateral modes form an orthogonal basis set such that the relative contributions to each guided mode from an input field distribution can be evaluated using an overlap integral between the input field profile and each mode.

Under the paraxial approximation, the propagation constants of the modes confined in the MMI region can be written as [7]

$$\beta_m \cong k_0 n - \frac{(m+1)^2 \pi \lambda_0}{4nW^2} \tag{2}$$

where $k_0$ is the wavenumber in free-space, $\lambda_0$ is the wavelength in free-space, $n$ is the effective refractive index of the transverse waveguide mode, and $W$ is the lateral width of the MMI region. By defining $L_\pi$ as the beat length of the two lowest order modes

$$L_\pi \equiv \frac{\pi}{\beta_0 - \beta_1} \cong \frac{4nW^2}{3\lambda_0} \tag{3}$$

the field profile at fixed positions within the MMI region can be written as [5]

$$\Psi\left(y,\, 3L_\pi \frac{M}{N}\right) = \frac{1}{C} \sum_{q=0}^{N-1} \Psi(y - y_q,\, 0) \exp(j\theta_q) \tag{4}$$

where $M$ and $N$ are integers without a common divisor and $C$ is a complex normalization constant. The form of (4) illustrates Talbot self-imaging. At a fixed distance $z = 3L_\pi(M/N)$, the output field profile is a sum of $N$-equal amplitude images of the input field distribution. The integer $M$ represents imaging periodicity along the $z$ propagation axis.

If the input to a $1 \times N$ splitter center-feeds the MMI region, only the even symmetric modes will be excited. Under this condition, a compact $1 \times N$ splitter can be realized with an MMI length of

$$z = \frac{1}{N} \cdot \left(\frac{3L_\pi}{4}\right). \tag{5}$$

Note that $M = 1$ in this case and symmetric interference reduces the MMI length by a factor of four.

## III. Simulation of Quasi-Optical Power Splitters and Combiners

We designed and simulated quasioptical power splitters and combiners for operation at 94 GHz. The rectangular waveguides used in this design are metal-clad and quartz-filled. Quartz is a positive uniaxial crystal and at 94 GHz it has an extraordinary index of refraction $n_e = 2.1366$ and an ordinary index of refraction $n_o = 2.0801$. The optic axis of the quartz dielectric is oriented parallel to the $z$-axis so that the TE polarized mode in the waveguide can be described by $n_o$.

In the power splitter, a square cross section waveguide with inner dimensions 1.15 mm $\times$ 1.15 mm center-feeds a wider MMI region with inner dimensions 1.15 mm $\times$ 24 mm. The MMI region's rectangular waveguide supports a single transverse mode and 31 lateral modes. As shown in Fig. 2, a modal propagation analysis (MPA) [7] of the lateral MMI modes indicates that eight self-images of the input field profile occur

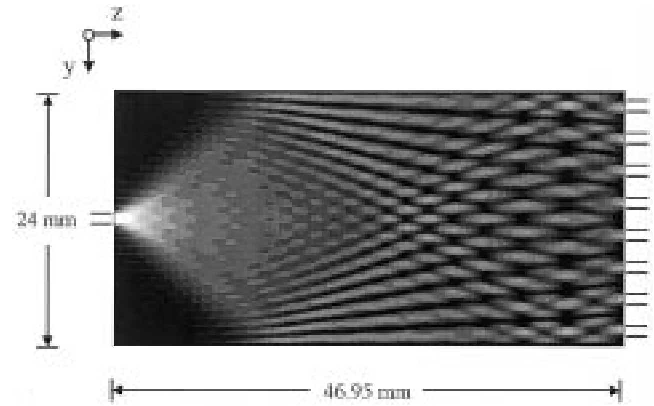


Fig. 2. Top view simulation of the electric field distribution of a $1 \times 8$ power splitter.

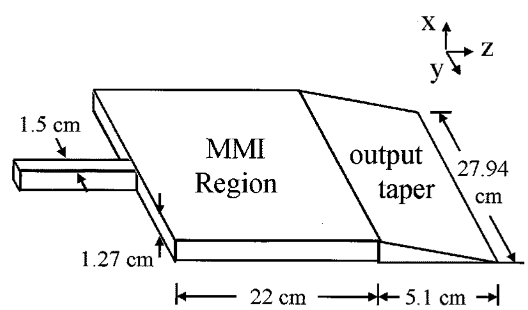


Fig. 3. Experimental setup used to demonstrate the multimode interference phenomenon at 8.231 GHz.

at an MMI region length of 46.95 mm. This simulation result compares favorably with the analytically obtained MMI length of 46.93 mm as predicted by (5). Less than 0.32 dB of power output nonuniformity among the 8 outputs is predicted. The 3 dB bandwidth is limited by the wavelength dependence of the Talbot effect and is 4.6% for our splitter.

To achieve a compact $8 \times 1$ power combiner using Talbot self-imaging, the same MMI dimensions as for the $1 \times 8$ splitter can be used. However, the relative phases among the 8 inputs must be tuned to $\{(\pi/2), (-3\pi/4), (-\pi/4), 0, 0, (-\pi/4), (-3\pi/4), (\pi/2)\}$. This input phase requirement insures that the MMI modes constructively interfere to efficiently couple into the single output waveguide. MPA simulation predicts that the output power distribution contains a centered main lobe with a 3 dB width of 0.53 mm. The peak sidelobe is $-36.28$ dB below the main lobe.

## IV. Experimental Results

To demonstrate the phenomenon of multimode interference in a quasioptical waveguide, the structure depicted in Fig. 3 was constructed and characterized at 8.231 GHz. The dielectric core material is a microwave plastic with the tradename Rexolite® 1422. It has a dielectric constant of 2.53 through 500 GHz. To reduce loss, the dielectric of both the input waveguide and the MMI region were wrapped with aluminum foil.

The MMI region was center-fed and a 5-cm-long tapered waveguide was used at the output of the MMI region to reduce reflection and to outcouple the energy. A 1-cm-long monopole antenna was scanned at 5 mm intervals across the lateral width

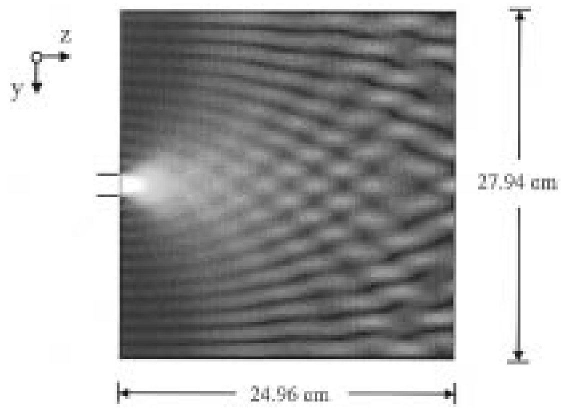


Fig. 4. Top view simulation of the electric field distribution in the Rexolite® waveguide at 8.231 GHz.

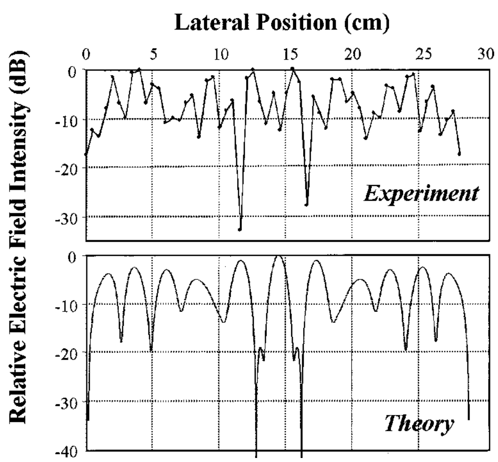


Fig. 5. Experimental and simulated data of the electric field profile at $z = 24.96$ cm. The refractive index used for Rexolite® was 1.61.

of the output taper. The signal was detected using a Hewlett-Packard 8510 Vector Network Analyzer.

Fig. 4 shows a simulation result of the system depicted in Fig. 3 with corresponding experimental data shown in Fig. 5. The MMI structure supports 24 lateral modes. The modal interference pattern is apparent in the simulation of Fig. 4 and verified by the presence and symmetry of the 2 nulls in the data of Fig. 5. The slight discrepancy in the position of the nulls is not adequately accounted by variations in MMI width and dielectric constant and is currently under investigation.

## V. Conclusion

The Talbot effect was investigated for quasioptical power splitting and combining. Talbot image formation in planar waveguides is based on the phenomenon of multimode interference. For the first time, we have demonstrated multimode interference in a rectangular waveguide operating at microwave frequencies. The use of Talbot imaging in a waveguide would obviate the need for lensing elements within the waveguide cavity—thereby decreasing the throughput losses. We have presented the design of a 1 × 8 power splitter and an 8 × 1 power combiner for operation at 94 GHz. In summary, the Talbot effect is a viable technology for quasioptical splitters and combiners in over-moded guided wave structures.

## Acknowledgment

The authors would like to thank Dr. D. M. Mackie for permission to use the *MMICalc* software used in the device simulations. They would also like to thank Dr. M. E. Thomas for providing refractive index data on quartz at 94 GHz.

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